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Supplementary material

Appendix 1

Table A1. Random effects retained in the final models (shown as "+") for von Bertalanffy, Gompertz, logistic, U_4 and Richards's growth models in body mass, bill length, head length and tarsus length. *A*, *k*, *Ti* and *d* are the upper asymptote, maximum relative growth rate, age at the inflection point and shape parameter, respectively. Prior to model comparisons (based on AICc), we evaluated the significance of random effects using likelihood ratio tests, with non-significant random effects ($p > 0.05$) being discarded. NA: not applicable.

		Random effects at the intercept			
Trait	Model	*A*	*k*	T_i	*d*
Body mass	von Bertalanffy	+			NA
Body mass	Gompertz	+			NA
Body mass	Logistic	+		+	NA
Body mass	U_4	+			NA
Body mass	Richards	+		+	
Bill length	von Bertalanffy	+		+	NA
Bill length	Gompertz	+		+	NA
Bill length	Logistic	+	+	+	NA
Bill length	U_4	+	+	+	NA
Bill length	Richards	+	+	+	
Headh length	von Bertalanffy		+	+	NA
Headh length	Gompertz		+	+	NA
Headh length	Logistic	+		+	NA
Headh length	U_4	+		+	NA
Headh length	Richards	+		+	
Tarsus length	von Bertalanffy				NA
Tarsus length	Gompertz			+	NA
Tarsus length	Logistic	+		+	NA
Tarsus length	U_4	+	+	+	NA
Tarsus length	Richards	+	+	+	

Appendix 2. R code for von Bertalanffy, Gompertz, logistic, U_4 and Richards's growth models using the Unified parameterization proposed by Tjørve & Tjørve (2017).

```
############################## Growth models ##############################
######################### Parameterizations Ti-form ########################

# A = upper asymptote
# k = maximum relative growth rate
# Ti = age at the inflection point
# d = shape parameter
# t = Age of the chick

# Unified parameterizations in the Ti-form from Tjørve & Tjørve (2017),
# Ecological Modelling 359: 117-127.

# von Bertalanffy: A*(1-((1/3)*exp(-(9/4)*k*(t-Ti))))^3
# Model (9) in Tjørve & Tjørve (2017)

# Gompertz: A*(exp(-exp(-exp(1)*k*(t-Ti))))
# Model (7) in Tjørve & Tjørve (2017)

# Logistic: A/(1+exp(-4*k*(t-Ti)))
# Model (5) in Tjørve & Tjørve (2017)

# U4: A*(1+(3*exp(-(4^(4/3))*k*(t-Ti))))^(-1/3)
# Model (11) in Tjørve & Tjørve (2017)

# Richards: A*(1+(d-1)*exp((-k*(t-Ti))/(d^(d/(1-d)))))^(1/(1-d))
# Model (3) in Tjørve & Tjørve (2017)

######################## Models on Instantaneous growth ####################
######################### Parameterizations Ti-form ########################
######### Equations were obtained as first derivatives of growth models ############

# von Bertalanffy:
# (9/4)*A*k*(exp(-(9/4)*k*(t-Ti)))*(1-((1/3)*(exp(-(9/4)*k*(t-Ti)))))^2

# Gompertz:
# exp(1)*A*k*(exp(-exp(1)*k*(t-Ti)))*(exp(-exp(-exp(1)*k*(t-Ti))))

# Logistic:
# 4*A*k*((exp(-4*k*(t-Ti)))/(1+exp(-4*k*(t-Ti)))^2)

# U4:
# (4^(4/3))*A*k*(exp((-4^(4/3))*k*(t-Ti)))*((1+(3*exp((-4^(4/3))*k*(t-Ti))))^(-4/3))

# Richards:
# ((1-d)/((1-d)*(d^(d/(1-d)))))*A*k*(exp(-k*(t-Ti)/(d^(d/(1-d)))))*((1+((d-1)*(exp(-k*(t-
Ti)/(d^(d/(1-d)))))))^(d/(1-d)))
```

```
###################### Maximum absolute growth rate ######################
# gmax = maximum absolute growth rate (gmax = A*k)


# Some examples
library(nlme)
RGM$t<-as.integer(RGM$t) # t is the age of the chick when measured
RGM$ID<-as.factor(RGM$ID) # Chick identity
RGM$BillLength<-as.numeric(RGM$BillLength) # Bill length data
RGM$BodyMass<-as.numeric(RGM$BodyMass) # Body mass data
RGM<-groupedData(BodyMass~t|ID,data=RGM) # Main structure as grouped data
VarFunc.Auto<-varPower(form=~fitted(.)) # Power variance function -for models in body
mass exhibiting heteroscedasticity-.

# Example of a Gompertz model in Bill length with random effects for A, k and Ti:
BL.G.1<-nlme(BillLength~A*(exp(-exp(-exp(1)*k*(t-Ti)))), fixed=list(A~1,k~1,Ti~1),
             random=list(ID=pdDiag(A+k+Ti~1)),
             data=RGM, start=c(A=66,k=0.022,Ti=12),
             method="ML",na.action=na.pass)
summary(BL.G.1)
logLik(BL.G.1)

# Example of a Gompertz model in Bill length with random effects for A and Ti:
BL.G.2<-nlme(BillLength~A*(exp(-exp(-exp(1)*k*(t-Ti)))), fixed=list(A~1,k~1,Ti~1),
            random=list(ID=pdDiag(A+Ti~1)),
            data=RGM, start=c(A=66,k=0.022,Ti=12),
            method="ML", na.action=na.pass)
summary(BL.G.2)
logLik(BL.G.2)
anova(BL.G.1,BL.G.2)

# Example of a Richards model in Body mass with a correction for heteroscedasticity and
random effects for A and Ti:
BM.R<-nlme(BodyMass~A*(1+(d-1)*exp((-k*(t-Ti))/(d^(d/(1-d)))))^(1/(1-d)),
           fixed=list(A~1,k~1,Ti~1,d~1),
          random=list(ID=pdDiag(A+Ti~1)),
          data=RGM, start=c(A=1866,k=0.043,Ti=17.2,d=1.55),
          weights=VarFunc.Auto, method="ML", na.action=na.pass)
summary(BM.R)
logLik(BM.R)
```